# INTERNATIONAL STANDARD 

## Quantities and units -

## Part 2: <br> Mathematics

Grandeurs et unités -
Partie 2: Mathématiques


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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 12, Quantities and units, in collaboration with Technical Committee IEC/TC 25, Quantities and units.

This second edition cancels and replaces the first edition (ISO 80000-2:2009), which has been technically revised.

The main changes compared to the previous edition are as follows:

- Clause 4 revised to add clarification about writing of font types; revised rule for splitting equations over two or more lines;
- Clause 18 revised to include clarification on scalars, vectors and tensors;
- missing symbols and expressions added in the second column "Symbol, expression" of the tables, and additional clarifications given in the fourth column "Remarks and examples" when necessary;
- Annex A deleted.

NOTE Although missing symbols and expressions have been added in this second edition of ISO 80000-1, the document remains non exhaustive.

A list of all parts in the ISO 80000 and IEC 80000 series can be found on the ISO and IEC websites.
Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

This corrected version of ISO 80000-2:2019 incorporates the following corrections:

— in 2-12.20, under "Remarks and examples", last line, $f^{\prime \prime \prime}$ has been replaced with $f^{\prime \prime}$;

- in 2-13.1, under "Remarks and examples", the value $2,7188128 \ldots$ has been replaced with 2,718 281828 ...;
- in 2-20.20, under "Remarks and examples", the first formula has been corrected to read $\mathrm{L}_{n}^{m}(z)=(-1)^{m} \frac{\mathrm{~d}^{m}}{\mathrm{~d} z^{m}} \mathrm{~L}_{n+m}(z)$; i.e. addition of $+m$ in the subscript of L ;
- in 2-20.21, under "Remarks and examples", second line, the parenthesis has been corrected to read (for $n \in \mathbf{N},|z| \leq 1$ ); i.e. addition of $|z| \leq 1$.


## Introduction

## Arrangement of the tables

Each table of symbols and expressions (except Table 13) gives hints (in the third column) about the meaning or how the expression may be read for each item (numbered in the first column) of the symbol under consideration, usually in the context of a typical expression (second column). If more than one symbol or expression is given for the same item, they are on an equal footing. In some cases, e.g. for exponentiation, there is only a typical expression and no symbol. The purpose of the entries is identification of each concept and is not intended to be a complete mathematical definition. The fourth column "Remarks and examples" gives further information and is not normative.

Table 13 has a different format. It gives the symbols of coordinates, as well as the position vectors and their differentials, for coordinate systems in three-dimensional spaces.

## Quantities and units -

## Part 2: <br> Mathematics

## 1 Scope

This document specifies mathematical symbols, explains their meanings, and gives verbal equivalents and applications.

This document is intended mainly for use in the natural sciences and technology, but also applies to other areas where mathematics is used.

## 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 80000-1, Quantities and units - Part 1: General

## 3 Terms and definitions

Tables 1 to 16 give the symbols and expressions used in the different fields of mathematics.
ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at https://www.iso.org/obp
- IEC Electropedia: available at http://www.electropedia.org/


## 4 Variables, functions and operators

It is customary to use different sorts of letters for different sorts of entities, e.g. $x, y, \ldots$ for numbers or elements of some given set, $f, g$ for functions, etc. This makes formulas more readable and helps in setting up an appropriate context.

Variables such as $x, y$, etc., and running numbers, such as $i$ in $\sum_{i} x_{i}$ are printed in italic type. Parameters, such as $a, b$, etc., which may be considered as constant in a particular context, are printed in italic type. The same applies to functions in general, e.g. $f, g$.

An explicitly defined function not depending on the context is, however, printed in upright type, e.g. sin, exp, $\ln$, . Mathematical constants, the values of which never change, are printed in upright type, e.g. e $=2,718281828 \ldots ; \pi=3,141592 \ldots \mathrm{i}^{2}=-1$. Well-defined operators are also printed in upright type, e.g. div, $\delta$ in $\delta x$ and each din $\mathrm{d} f / \mathrm{d} x$. Some transforms use special capital letters (see Clause 19, Transforms).

Numbers expressed in the form of digits are always printed in upright type, e.g. 351 204; 1,32; 7/8.
Binary operators, for example,+- , /, shall be preceded and followed by thin spaces. This rule does not apply in case of unary operators, as in $-17,3$.

