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**Control charts —**

**Part 4:  
Cumulative sum charts**

*Cartes de contrôle —*

*Partie 4: Cartes de contrôle à somme cumulée*

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see [www.iso.org/iso/foreword.html](http://www.iso.org/iso/foreword.html).

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This second edition of ISO 7870-4 cancels and replaces the first edition (ISO 7870-4: 2011), which has been technical revised.

The main changes compared to the previous edition are as follows:

- Manhattan diagram removed (former 6.7);
- V-mask types in Types of CUSUM decision schemes reduced to one V-mask;
- von Neumann method removed (former Annex A).

A list of all parts in the ISO 7870 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at [www.iso.org/members.html](http://www.iso.org/members.html).

## Introduction

This document demonstrates the versatility and usefulness of a very simple, yet powerful, pictorial method of interpreting data arranged in any meaningful sequence. These data can range from overall business figures such as turnover, profit or overheads to detailed operational data such as stock outs and absenteeism to the control of individual process parameters and product characteristics. The data can either be expressed sequentially as individual values on a continuous scale (e.g. 24, 60, 31, 21, 18, 97...), in 'yes'/'no', 'good'/'bad', 'success'/'failure' format, or as summary measures (e.g. mean, range, counts of events).

The method has a rather unusual name, cumulative sum, or CUSUM. This name relates to the process of subtracting a predetermined value, e.g. a target, preferred or reference value from each observation in a sequence and progressively cumulating (i.e. adding) the differences. The graph of the series of cumulative differences is known as a CUSUM chart. Such a simple arithmetical process has a remarkable effect on the visual interpretation of the data.

The CUSUM method is already used unwittingly by golfers throughout the world. By scoring a round as 'plus' 4, or perhaps even 'minus' 2, golfers are using the CUSUM method in a numerical sense. They subtract the 'par' value from their actual score and add (cumulate) the resulting differences. This is the CUSUM method in action. However, it remains largely unknown and hence is a grossly underused tool throughout business, industry, commerce and public service. This is probably due to CUSUM methods generally being presented in statistical language rather than in the language of the workplace.

The intention of this document is, thus, to be readily comprehensible to the extensive range of prospective users and so facilitate widespread communication and understanding of the method. The method offers advantages over the more commonly found Shewhart charts in as much as the CUSUM method detects a change of an important amount up to three times faster. Further, as in golf, when the target changes per hole, a CUSUM plot is unaffected, unlike a standard Shewhart chart where the control lines require constant adjustment.

In addition to Shewhart charts, an EWMA (exponentially weighted moving average) chart can be used. Each plotted point on an EWMA chart incorporates information from all the previous subgroups or observations but gives less weight to process data as they get 'older' according to an exponentially decaying weight. In a similar manner to a CUSUM chart, an EWMA chart can be sensitized to detect any size of shift in a process. This subject is discussed further in 7870-6.

# Control charts —

## Part 4: Cumulative sum charts

### 1 Scope

This document describes statistical procedures for setting up cumulative sum (CUSUM) schemes for process and quality control using variables (measured) and attribute data. It describes general-purpose methods of decision-making using cumulative sum (CUSUM) techniques for monitoring, control and retrospective analysis.

### 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

### 3 Terms and definitions, abbreviated terms and symbols

For the purposes of this document, the terms and definitions given in ISO 3534-1 and ISO 3534-2 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

#### 3.1 Terms and definitions

##### 3.1.1

##### target value

$T$

value for which a departure from an average level is required to be detected

Note 1 to entry: With a charted CUSUM, the deviations from the target value are cumulated.

Note 2 to entry: Using a 'V' mask, the target value is often referred to as the reference value or the nominal control value. If so, it needs be acknowledged that it is not necessarily the most desirable or preferred value, as can appear in other standards. It is simply a convenient target value for constructing a CUSUM chart.

##### 3.1.2

##### representative out of control value

(tabulated CUSUM) value which controls the sensitivity of the procedure

Note 1 to entry: The upper out of control value is  $T + f\sigma_e$ , for monitoring an upward shift. The lower control value is  $T - f\sigma_e$ , for monitoring a downward shift.