

Mathematical expressions for reliability, availability, maintainability and maintenance support terms

## EESTI STANDARDI EESSÕNA

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English Version

Mathematical expressions for reliability, availability,  
maintainability and maintenance support terms  
(IEC 61703:2016)

Expressions mathématiques pour les termes de fiabilité, de  
disponibilité, de maintenabilité et de logistique de  
maintenance  
(IEC 61703:2016)

Mathematische Ausdrücke für Begriffe der Zuverlässigkeit,  
Verfügbarkeit, Instandhaltbarkeit und  
Instandhaltungsbereitschaft  
(IEC 61703:2016)

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## European foreword

The text of document 56/1682/FDIS, future edition 2 of IEC 61703, prepared by IEC/TC 56 "Dependability" was submitted to the IEC-CENELEC parallel vote and approved by CENELEC as EN 61703:2016.

The following dates are fixed:

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publication of an identical national  
standard or by endorsement
- latest date by which the national (dow) 2019-09-16  
standards conflicting with the  
document have to be withdrawn

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In the official version, for Bibliography, the following notes have to be added for the standards indicated:

IEC 61508 Series	NOTE	Harmonized as EN 61508 Series.
IEC 61511 Series	NOTE	Harmonized as EN 61511 Series.
IEC 61025	NOTE	Harmonized as EN 61025.
IEC 61078	NOTE	Harmonized as EN 61078.
IEC 61165	NOTE	Harmonized as EN 61165.

## Annex ZA (normative)

### Normative references to international publications with their corresponding European publications

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE 1 When an International Publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

NOTE 2 Up-to-date information on the latest versions of the European Standards listed in this annex is available here: [www.cenelec.eu](http://www.cenelec.eu)

<u>Publication</u>	<u>Year</u>	<u>Title</u>	<u>EN/HD</u>	<u>Year</u>
IEC 60050-192	2015	International electrotechnical vocabulary - Part 192: Dependability	-	-
ISO 3534-1	2006	Statistics - Vocabulary and symbols - Part 1: General statistical terms and terms used in probability	-	-

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## INTRODUCTION

IEC 60050-192 provides definitions for dependability and its influencing factors, reliability, availability, maintainability and maintenance support, together with definitions of other related terms commonly used in this field. Some of these terms are measures of specific dependability characteristics, which can be expressed mathematically.

It is important for the users to understand the mathematical meaning of those expressions and how they are established. This is the purpose of the present International Standard which, used in conjunction with IEC 60050-192, provides practical guidance essential for the quantification of those measures. For those requiring further information, for example on detailed statistical methods, reference should be made to the IEC 60605 series [23]<sup>1</sup>.

Annex A provides a diagrammatic explanation of the relationships between some basic dependability terms, related random variables, probabilistic descriptors and modifiers.

Annex B provides a summary of measures related to time to failure.

Annex C compares some dependability measures for continuously operating items.

The bibliography gives references for the mathematical basis of this standard, in particular, the mathematical material is based on references [2], [6], [8], [9], [13], [14] and [18]; the renewal theory (renewal and alternating renewal processes) can be found in [6], [8], [9], [10], [11], [13], [15], and [17]; and more advanced treatment of renewal theory can be found in references [1], [3], [12], [16], [19] and [20]. More information on the theory and applications of Markov processes can be found in references [3], [9], [10], [15], [16], [17] and [19].

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<sup>1</sup> Numbers in brackets refer to the Bibliography.

## MATHEMATICAL EXPRESSIONS FOR RELIABILITY, AVAILABILITY, MAINTAINABILITY AND MAINTENANCE SUPPORT TERMS

### 1 Scope

This International Standard provides mathematical expressions for selected reliability, availability, maintainability and maintenance support measures defined in IEC 60050-192:2015. In addition, it introduces some terms not covered in IEC 60050-192:2015. They are related to aspects of the system of item classes (see hereafter).

According to IEC 60050-192:2015, dependability [192-01-22] is the ability of an item to perform as and when required and an item [192-01-01] can be an individual part, component, device, functional unit, equipment, subsystem, or system.

To account for mathematical constraints, this standard splits the items between the individual items considered as a whole (e.g. individual components) and the systems made of several individual items. It provides general considerations for the mathematical expressions for systems as well as individual items but the individual items which are easier to model are analysed in more detail with regards to their repair aspects.

The following item classes are considered separately:

- Systems;
- Individual items:
  - non-repairable [192-01-12];
  - repairable [192-01-11]:
    - i) with zero (or negligible) time to restoration;
    - ii) with non-zero time to restoration.

In order to explain the dependability concepts which can be difficult to understand, keep the standard self-contained and the mathematical formulae as simple as possible, the following basic mathematical models are used in this standard to quantify dependability measures:

- Systems:
  - state-transition models;
  - Markovian models.
- Individual items:
  - random variable (time to failure) for non-repairable items;
  - simple (ordinary) renewal process for repairable items with zero time to restoration;
  - simple (ordinary) alternating renewal process for repairable items with non-zero time to restoration.

The application of each dependability measure is illustrated by means of simple examples.

This standard is mainly applicable to hardware dependability, but many terms and their definitions may be applied to items containing software.