

## Power law model - Goodness-of-fit tests and estimation methods

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**Power law model -  
Goodness-of-fit tests and estimation methods  
(IEC 61710:2013)**

Modèle de loi en puissance -  
Essais d'adéquation et méthodes  
d'estimation des paramètres  
(CEI 61710:2013)

Potenzgesetz-Modell -  
Anpassungstests und Schätzverfahren  
(IEC 61710:2013)

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Europäisches Komitee für Elektrotechnische Normung

**CEN-CENELEC Management Centre: Avenue Marnix 17, B - 1000 Brussels**

## Foreword

The text of document 56/1500/FDIS, future edition 2 of IEC 61710, prepared by IEC/TC 56 "Dependability" was submitted to the IEC-CENELEC parallel vote and approved by CENELEC as EN 61710:2013.

The following dates are fixed:

- latest date by which the document has to be (dop) 2014-03-26  
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IEC 61703	NOTE	Harmonised as EN 61703.
IEC 61164:2004	NOTE	Harmonised as EN 61164:2004 (not modified).

## **Annex ZA** (normative)

### **Normative references to international publications with their corresponding European publications**

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

NOTE When an international publication has been modified by common modifications, indicated by (mod), the relevant EN/HD applies.

<u>Publication</u>	<u>Year</u>	<u>Title</u>	<u>EN/HD</u>	<u>Year</u>
IEC 60050-191	1990	International Electrotechnical Vocabulary (IEV) - Chapter 191: Dependability and quality of service	-	-

## CONTENTS

FOREWORD.....	5
INTRODUCTION.....	7
1 Scope.....	8
2 Normative references .....	8
3 Terms and definitions .....	8
4 Symbols and abbreviations.....	8
5 Power law model .....	9
6 Data requirements .....	10
6.1 General.....	10
6.1.1 Case 1 – Time data for every relevant failure for one or more copies from the same population .....	10
6.1.2 Case 1a) – One repairable item .....	10
6.1.3 Case 1b) – Multiple items of the same kind of repairable item observed for the same length of time .....	11
6.1.4 Case 1c) – Multiple repairable items of the same kind observed for different lengths of time .....	11
6.2 Case 2 – Time data for groups of relevant failures for one or more repairable items from the same population.....	12
6.3 Case 3 – Time data for every relevant failure for more than one repairable item from different populations .....	12
7 Statistical estimation and test procedures.....	13
7.1 Overview .....	13
7.2 Point estimation .....	13
7.2.1 Case 1a) and 1b) – Time data for every relevant failure.....	13
7.2.2 Case 1c) – Time data for every relevant failure.....	14
7.2.3 Case 2 – Time data for groups of relevant failures.....	15
7.3 Goodness-of-fit tests .....	16
7.3.1 Case 1 – Time data for every relevant failure.....	16
7.3.2 Case 2 – Time data for groups of relevant failures.....	17
7.4 Confidence intervals for the shape parameter.....	18
7.4.1 Case 1 – Time data for every relevant failure.....	18
7.4.2 Case 2 – Time data for groups of relevant failures.....	19
7.5 Confidence intervals for the failure intensity .....	20
7.5.1 Case 1 – Time data for every relevant failure.....	20
7.5.2 Case 2 – Time data for groups of relevant failures.....	20
7.6 Prediction intervals for the length of time to future failures of a single item.....	21
7.6.1 Prediction interval for length of time to next failure for case 1 – Time data for every relevant failure .....	21
7.6.2 Prediction interval for length of time to $R$ th future failure for case 1 – Time data for every relevant failure .....	22
7.7 Test for the equality of the shape parameters $\beta_1, \beta_2, \dots, \beta_k$ .....	23
7.7.1 Case 3 – Time data for every relevant failure for two items from different populations .....	23
7.7.2 Case 3 – Time data for every relevant failure for three or more items from different populations .....	24
Annex A (informative) The power law model – Background information.....	30
Annex B (informative) Numerical examples.....	31

Annex C (informative) Bayesian estimation for the power law model .....	41
Bibliography .....	56
Figure 1 – One repairable item .....	10
Figure 2 – Multiple items of the same kind of repairable item observed for same length of time .....	11
Figure 3 – Multiple repairable items of the same kind observed for different lengths of time .....	12
Figure B.1 – Accumulated number of failures against accumulated time for software system .....	32
Figure B.2 – Expected against observed accumulated times to failure for software system .....	32
Figure B.3 – Accumulated number of failures against accumulated time for five copies of a system .....	35
Figure B.4 – Accumulated number of failures against accumulated time for an OEM product from vendors A and B .....	37
Figure B.5 – Accumulated number of failures against time for generators .....	38
Figure B.6 – Expected against observed accumulated number of failures for generators .....	39
Figure C.1 – Plot of fitted Gamma prior (6,7956, 0,0448) for the shape parameter of the power law model .....	47
Figure C.2 – Plot of fitted Gamma prior (17,756 6, 1447,408) for the expected number of failures parameter of the power law model .....	47
Figure C.3 – Subjective distribution of number of failures .....	51
Figure C.4 – Plot of the posterior probability distribution for the number of future failures, $M$ .....	54
Figure C.5 – Plot of the posterior cumulative distribution for the number of future failures, $M$ .....	55
Table 1 – Critical values for Cramer-von-Mises goodness-of-fit test at 10 % level of significance .....	25
Table 2 – Fractiles of the Chi-square distribution .....	26
Table 3 – Multipliers for two-sided 90 % confidence intervals for intensity function for time terminated data .....	27
Table 4 – Multipliers for two-sided 90 % confidence intervals for intensity function for failure terminated data .....	28
Table 5 – 0,95 fractiles of the $F$ distribution .....	29
Table B.1 – All relevant failures and accumulated times for software system .....	31
Table B.2 – Calculation of expected accumulated times to failure for Figure B.2 .....	33
Table B.3 – Accumulated times for all relevant failures for five copies of a system (labelled A, B, C, D, E) .....	34
Table B.4 – Combined accumulated times for multiple items of the same kind of a system .....	34
Table B.5 – Accumulated operating hours to failure for OEM product from vendors A and B .....	36
Table B.6 – Grouped failure data for generators .....	38
Table B.7 – Calculation of expected numbers of failures for Figure B.6 .....	40
Table C.1 – Strengths and weakness of classical and Bayesian estimation .....	42

Table C.2 – Grid for eliciting subjective distribution for shape parameter $\beta$ .....	46
Table C.3 – Grid for eliciting subjective distribution for expected number of failures parameter $\eta$ .....	46
Table C.4 – Comparison of fitted Gamma and subjective distribution for shape parameter $\beta$ .....	48
Table C.5 – Comparison of fitted Gamma and subjective distribution for expected number of failures by time $T = 20\,000$ h parameter $\eta$ .....	48
Table C.6 – Times to failure data collected on system test.....	49
Table C.7 – Summary of estimates of power law model parameters .....	50
Table C.8 – Time to failure data for operational system .....	53

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## INTRODUCTION

This International Standard describes the power law model and gives step-by-step directions for its use. There are various models for describing the reliability of repairable items, the power law model being one of the most widely used. This standard provides procedures to estimate the parameters of the power law model and to test the goodness-of-fit of the power law model to data, to provide confidence intervals for the failure intensity and prediction intervals for the length of time to future failures. An input is required consisting of a data set of times at which relevant failures occurred, or were observed, for a repairable item or a set of copies of the same item, and the time at which observation of the item was terminated, if different from the time of final failure. All output results correspond to the item type under consideration.

Some of the procedures can require computer programs, but these are not unduly complex. This standard presents algorithms from which computer programs should be easy to construct.

# POWER LAW MODEL – GOODNESS-OF-FIT TESTS AND ESTIMATION METHODS

## 1 Scope

This International Standard specifies procedures to estimate the parameters of the power law model, to provide confidence intervals for the failure intensity, to provide prediction intervals for the times to future failures, and to test the goodness-of-fit of the power law model to data from repairable items. It is assumed that the time to failure data have been collected from an item, or some identical items operating under the same conditions (e.g. environment and load).

## 2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

IEC 60050-191:1990, *International Electrotechnical Vocabulary (IEV) – Chapter 191: Dependability and quality of service*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions of IEC 60050-191 apply.

## 4 Symbols and abbreviations

The following symbols and abbreviations apply:

$\beta$	shape parameter of the power law model
$\hat{\beta}$	estimated shape parameter of the power law model
$\beta_{LB}, \beta_{UB}$	lower, upper confidence limits for $\beta$
$C^2$	Cramer-von-Mises goodness-of-fit test statistic
$C^2_{1-\gamma}(M)$	critical value for the Cramer-von-Mises goodness-of-fit test statistic at $\gamma$ level of significance
$\chi^2$	Chi-square goodness-of-fit test statistic
$\chi^2_{\gamma}(v)$	$\gamma$ th fractile of the $\chi^2$ distribution with $v$ degrees of freedom
$d$	number of intervals for groups of failures
$E[N(t)]$	expected accumulated number of failures up to time $t$
$E[t_j]$	expected accumulated time to $j$ th failure