

---

---

**Control charts —**

**Part 4:  
Cumulative sum charts**

*Cartes de contrôle —*

*Partie 4: Cartes de contrôle de l'ajustement de processus*



This document is a preview generated by EVS



**COPYRIGHT PROTECTED DOCUMENT**

© ISO 2011

All rights reserved. Unless otherwise specified, no part of this publication may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying and microfilm, without permission in writing from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office  
Case postale 56 • CH-1211 Geneva 20  
Tel. + 41 22 749 01 11  
Fax + 41 22 749 09 47  
E-mail [copyright@iso.org](mailto:copyright@iso.org)  
Web [www.iso.org](http://www.iso.org)

Published in Switzerland

# Contents

Page

Foreword .....	iv
Introduction.....	v
1 Scope .....	1
2 Normative references .....	1
3 Terms and definitions, abbreviated terms and symbols.....	1
3.1 Terms and definitions .....	1
3.2 Abbreviated terms .....	2
3.3 Symbols.....	3
4 Principal features of cumulative sum (cusum) charts.....	4
5 Basic steps in the construction of cusum charts — Graphical representation.....	5
6 Example of a cusum plot — Motor voltages.....	5
6.1 The process .....	5
6.2 Simple plot of results .....	6
6.3 Standard control chart for individual results .....	7
6.4 Cusum chart — Overall perspective.....	7
6.5 Cusum chart construction.....	8
6.6 Cusum chart interpretation .....	9
6.7 Manhattan diagram.....	12
7 Fundamentals of making cusum-based decisions .....	12
7.1 The need for decision rules.....	12
7.2 The basis for making decisions .....	13
7.3 Measuring the effectiveness of a decision rule.....	14
8 Types of cusum decision schemes .....	16
8.1 V-mask types .....	16
8.2 Truncated V-mask .....	16
8.3 Alternative design approaches .....	22
8.4 Semi-parabolic V-mask .....	23
8.5 Snub-nosed V-mask .....	24
8.6 Full V-mask .....	24
8.7 Fast initial response (FIR) cusum .....	25
8.8 Tabular cusum .....	25
9 Cusum methods for process and quality control .....	27
9.1 The nature of the changes to be detected .....	27
9.2 Selecting target values .....	28
9.3 Cusum schemes for monitoring location .....	29
9.4 Cusum schemes for monitoring variation .....	39
9.5 Special situations .....	47
9.6 Cusum schemes for discrete data.....	49
Annex A (informative) Von Neumann method.....	56
Annex B (informative) Example of tabular cusum .....	57
Annex C (informative) Estimation of the change point when a step change occurs.....	61
Bibliography.....	63

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 7870-4 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

This first edition of ISO 7870-4 cancels and replaces ISO/TR 7871:1997.

ISO 7870 consists of the following parts, under the general title *Control charts*:

- *Part 1: General guidelines*
- *Part 3: Acceptance control charts*
- *Part 4: Cumulative sum charts*

The following part is under preparation:

- *Part 2: Shewhart control charts*

Additional parts on specialized control charts and on the application of statistical process control (SPC) charts are planned.

## Introduction

This part of ISO 7870 demonstrates the versatility and usefulness of a very simple, yet powerful, pictorial method of interpreting data arranged in any meaningful sequence. These data can range from overall business figures such as turnover, profit or overheads to detailed operational data such as stock outs and absenteeism to the control of individual process parameters and product characteristics. The data can either be expressed sequentially as individual values on a continuous scale (e.g. 24,60, 31,21, 18,97...), in “yes”/“no”, “good”/“bad”, “success”/“failure” format, or as summary measures (e.g. mean, range, counts of events).

The method has a rather unusual name, cumulative sum, or, in short, “cusum”. This name relates to the process of subtracting a predetermined value, e.g. a target, preferred or reference value from each observation in a sequence and progressively cumulating (i.e. adding) the differences. The graph of the series of cumulative differences is known as a cusum chart. Such a simple arithmetical process has a remarkable effect on the visual interpretation of the data as will be illustrated.

The cusum method is already used unwittingly by golfers throughout the world. By scoring a round as “plus” 4, or perhaps even “minus” 2, golfers are using the cusum method in a numerical sense. They subtract the “par” value from their actual score and add (cumulate) the resulting differences. This is the cusum method in action. However, it remains largely unknown and hence is a grossly underused tool throughout business, industry, commerce and public service. This is probably due to cusum methods generally being presented in statistical language rather than in the language of the workplace.

This part of ISO 7870 is a revision of ISO/TR 7871:1997. The intention of this part is, thus, to be readily comprehensible to the extensive range of prospective users and so facilitate widespread communication and understanding of the method. The method offers advantages over the more commonly found Shewhart charts in as much as the cusum method will detect a change of an important amount up to three times faster. Further, as in golf, when the target changes per hole, a cusum plot is unaffected, unlike a standard Shewhart chart where the control lines would require a constant adjustment.

In addition to Shewhart charts, an EWMA (exponentially weighted moving average) chart, can be used. Each plotted point on an EWMA chart incorporates information from all of the previous subgroups or observations, but gives less weight to process data as they get “older” according to an exponentially decaying weight. In a similar manner to a cusum chart, an EWMA chart can be sensitized to detect any size of shift in a process. This subject is discussed further in another part of this International Standard.



# Control charts —

## Part 4: Cumulative sum charts

### 1 Scope

This part of ISO 7870 provides statistical procedures for setting up cumulative sum (cusum) schemes for process and quality control using variables (measured) and attribute data. It describes general-purpose methods of decision-making using cumulative sum (cusum) techniques for monitoring, control and retrospective analysis.

### 2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

### 3 Terms and definitions, abbreviated terms and symbols

For the purposes of this document, the terms and definitions given in ISO 3534-1 and ISO 3534-2 and the following apply.

#### 3.1 Terms and definitions

##### 3.1.1

##### target value

$T$

value for which a departure from an average level is required to be detected

NOTE 1 With a charted cusum, the deviations from the target value are cumulated.

NOTE 2 Using a “V” mask, the target value is often referred to as the reference value or the nominal control value. If so, it should be acknowledged that it is not necessarily the most desirable or preferred value, as may appear in other standards. It is simply a convenient target value for constructing a cusum chart.

##### 3.1.2

##### datum value

⟨tabulated cusum⟩ value from which differences are calculated

NOTE The upper datum value is  $T + f\sigma_e$ , for monitoring an upward shift. The lower datum value is  $T - f\sigma_e$ , for monitoring a downward shift.