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# INTERNATIONAL STANDARD

Information technology – Microprocessor Systems – Floating-Point arithmetic



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3, rue de Varembe  
CH-1211 Geneva 20  
Switzerland  
Email: [inmail@iec.ch](mailto:inmail@iec.ch)  
Web: [www.iec.ch](http://www.iec.ch)

The Institute of Electrical and Electronics Engineers, Inc  
3 Park Avenue  
US-New York, NY10016-5997  
USA  
Email: [stds-info@ieee.org](mailto:stds-info@ieee.org)  
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## Contents

1. Overview .....	1
1.1 Scope .....	1
1.2 Purpose .....	1
1.3 Inclusions .....	1
1.4 Exclusions .....	2
1.5 Programming environment considerations .....	2
1.6 Word usage .....	2
2. Definitions, abbreviations, and acronyms .....	3
2.1 Definitions .....	3
2.2 Abbreviations and acronyms .....	5
3. Floating-point formats .....	6
3.1 Overview .....	6
3.2 Specification levels .....	7
3.3 Sets of floating-point data .....	7
3.4 Binary interchange format encodings .....	9
3.5 Decimal interchange format encodings .....	10
3.6 Interchange format parameters .....	13
3.7 Extended and extendable precisions .....	14
4. Attributes and rounding .....	15
4.1 Attribute specification .....	15
4.2 Dynamic modes for attributes .....	15
4.3 Rounding-direction attributes .....	16
5. Operations .....	17
5.1 Overview .....	17
5.2 Decimal exponent calculation .....	18
5.3 Homogeneous general-computational operations .....	19
5.4 formatOf general-computational operations .....	21
5.5 Quiet-computational operations .....	23
5.6 Signaling-computational operations .....	24
5.7 Non-computational operations .....	24
5.8 Details of conversions from floating-point to integer formats .....	26
5.9 Details of operations to round a floating-point datum to integral value .....	27
5.10 Details of totalOrder predicate .....	28
5.11 Details of comparison predicates .....	29
5.12 Details of conversion between floating-point data and external character sequences .....	30
6. Infinity, NaNs, and sign bit .....	34
6.1 Infinity arithmetic .....	34
6.2 Operations with NaNs .....	34
6.3 The sign bit .....	35
7. Default exception handling .....	36
7.1 Overview: exceptions and flags .....	36
7.2 Invalid operation .....	37
7.3 Division by zero .....	37
7.4 Overflow .....	37
7.5 Underflow .....	38
7.6 Inexact .....	38
8. Alternate exception handling attributes .....	39
8.1 Overview .....	39
8.2 Resuming alternate exception handling attributes .....	39
8.3 Immediate and delayed alternate exception handling attributes .....	40

9. Recommended operations .....	41
9.1 Conforming language- and implementation-defined functions .....	41
9.2 Recommended correctly rounded functions .....	42
9.3 Operations on dynamic modes for attributes .....	46
9.4 Reduction operations .....	46
10. Expression evaluation .....	48
10.1 Expression evaluation rules .....	48
10.2 Assignments, parameters, and function values .....	48
10.3 preferredWidth attributes for expression evaluation .....	49
10.4 Literal meaning and value-changing optimizations .....	50
11. Reproducible floating-point results .....	51
Annex A (informative) Bibliography .....	53
Annex B (informative) Program debugging support .....	55
Index of operations .....	57

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This introduction is not part of IEEE Std 754-2008, IEEE Standard for Floating-Point Arithmetic.

This standard is a product of the Floating-Point Working Group of, and sponsored by, the Microprocessor Standards Committee of the IEEE Computer Society.

This standard provides a discipline for performing floating-point computation that yields results independent of whether the processing is done in hardware, software, or a combination of the two. For operations specified in the normative part of this standard, numerical results and exceptions are uniquely determined by the values of the input data, the operation, and the destination, all under user control.

This standard defines a family of commercially feasible ways for systems to perform binary and decimal floating-point arithmetic. Among the desiderata that guided the formulation of this standard were:

- a) Facilitate movement of existing programs from diverse computers to those that adhere to this standard as well as among those that adhere to this standard.
- b) Enhance the capabilities and safety available to users and programmers who, although not expert in numerical methods, might well be attempting to produce numerically sophisticated programs.
- c) Encourage experts to develop and distribute robust and efficient numerical programs that are portable, by way of minor editing and recompilation, onto any computer that conforms to this standard and possesses adequate capacity. Together with language controls it should be possible to write programs that produce identical results on all conforming systems.
- d) Provide direct support for
  - execution-time diagnosis of anomalies
  - smoother handling of exceptions
  - interval arithmetic at a reasonable cost.
- e) Provide for development of
  - standard elementary functions such as *exp* and *cos*
  - high precision (multiword) arithmetic
  - coupled numerical and symbolic algebraic computation.
- f) Enable rather than preclude further refinements and extensions.

In programming environments, this standard is also intended to form the basis for a dialog between the numerical community and programming language designers. It is hoped that language-defined methods for the control of expression evaluation and exceptions might be defined in coming years, so that it will be possible to write programs that produce identical results on all conforming systems. However, it is recognized that utility and safety in languages are sometimes antagonists, as are efficiency and portability.

Therefore, it is hoped that language designers will look on the full set of operation, precision, and exception controls described here as a guide to providing the programmer with the ability to portably control expressions and exceptions. It is also hoped that designers will be guided by this standard to provide extensions in a completely portable way.

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# INFORMATION TECHNOLOGY — MICROPROCESSOR SYSTEMS —

## Floating-Point arithmetic

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### 1. Overview

#### 1.1 Scope

This standard specifies formats and methods for floating-point arithmetic in computer systems—standard and extended functions with single, double, extended, and extendable precision—and recommends formats for data interchange. Exception conditions are defined and standard handling of these conditions is specified.

#### 1.2 Purpose

This standard provides a method for computation with floating-point numbers that will yield the same result whether the processing is done in hardware, software, or a combination of the two. The results of the computation will be identical, independent of implementation, given the same input data. Errors, and error conditions, in the mathematical processing will be reported in a consistent manner regardless of implementation.

#### 1.3 Inclusions

This standard specifies:

- Formats for binary and decimal floating-point data, for computation and data interchange.
- Addition, subtraction, multiplication, division, fused multiply add, square root, compare, and other operations.
- Conversions between integer and floating-point formats.
- Conversions between different floating-point formats.
- Conversions between floating-point formats and external representations as character sequences.
- Floating-point exceptions and their handling, including data that are not numbers (NaNs).