TECHNICAL REPORT

ISO/TR 7871

First edition
1997-02-15

Cumulative sum charts — Guidance on quality control and data analysis using CUSUM techniques

Cartes des sommes cumulées — Lignes directrices pour le contrôle de la qualité et l'analyse des données utilisant les procédures CUSUM

Reference number
ISO/TR 7871:1997(E)

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Printed in Switzerland

Foreword

ISO (the International Organisation for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental or non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards. In exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 7871, which is a Technical Report of type 3, was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Statistical process control*.

0 Introduction

0.1 Basis of cusum chart

The cumulative sum chart (hereafter referred to by the generally accepted contraction "cusum chart") is a highly informative graphical presentation of data which are ordered in a logical sequence. Frequently this sequence corresponds to the order of observation on a time scale.

A reference value, T, is subtracted from each observation. This reference value is generally a constant but may be a prediction from a forecasting model or a target which may vary. The cumulative sums of the deviations from T are formed, and these cusums (C) are plotted against the serial numbers of the observations.

In a cusum chart intended to check a process for departure from a mean value equal to the reference value, that value is also known as the target value or aim. Without more advanced cusum procedures the two concepts, target value or aim and reference value must be distinguished. The former refers to the actual or intended process average, the latter to the reference values used in the cusum procedure. The intuitive appeal of the term target value is strong, however, and for most of this standard, clauses 0 - 6, the common value of target value and reference value is referred to as target value when this does not create ambiguity. In clause 6 upper and lower reference values are created and these must be distinguished from target values or aims.

The cusum method of plotting results is the representation of average by the local slope of the chart. When the local average corresponds to the target value, the path of the cusum lies roughly parallel to the sequence axis. When the local average of the series is greater than the target value, the cusum slopes upwards ; conversely, when the local average is less than the target value, the cusum slopes downwards. The greater the discrepancy between the local average and the target value, the steeper the slope of the cusum path.

The result of plotting the cusum is that changes in average level over different subdivisions of the total sequence of observations are clearly indicated by changes in slope of the chart. The local averages in such subdivisions can be readily estimated, either from the numerical values of the cusum from which the chart is plotted or directly, from the chart itself.

A second effect of using cumulative sum procedures is that there is an inherent serial dependence between the successive cumulative sums. Decisions regarding acceptable departures from the sequence axis require the use of the method of stochastic processes.

0.2 Simple example of cusum chart

The above principles are best appreciated from a simple example. The calculations and plotting procedure will, at this point, be developed without mathematical symbolism.

It is supposed that the following individual observations have been obtained, over a time sequence in order shown, and that a reference value of 15 is appropriate.

Table 1 : Data for cusum plotting

Observation number	Observed value	Deviation from reference value (= 15)	Cumulative sum of deviations
1	12	- 3	- 3
2	17	+ 2	- 1
3	14	- 1	- 2
4	14	- 1	- 3
5	17	+ 2	- 1
6	16	+ 1	0
7	14	- 1	- 1
8	11	- 4	- 5
9	13	- 2	- 7
10	14	- 1	- 8
11	15	0	- 8
12	11	- 4	- 12
13	14	- 1	- 13
14	16	+ 1	- 12
15	13	- 2	- 14
16	14	- 1	- 15
17	11	- 4	- 19
18	12	- 3	- 22
19	13	- 2	- 24
20	16	+ 1	- 23
21	12	- 3	- 26
22	18	+ 3	- 23
23	18	+ 3	- 20
24	17	+ 2	- 18
25	20	+ 5	- 13
26	15	0	- 13
27	14	- 1	- 14
28	18	+ 3	- 11
29	20	+ 5	- 6
30	16	+ 1	- 5
31	18	+ 3	- 2
32	14	- 1	- 3
33	16	+ 1	- 2

For a conventional control chart, as in figure 1, the observed values are plotted against their corresponding observation numbers. There is some indication that the last dozen values appear to be clustered around a different mean level from the first 20 or so.

Plotting in the cusum mode give a much clearer display than the conventional chart. The cusum (column 4 of table 1) is plotted against the observation number using the y ("vertical") axis for the cusum and the x ("horizontal") axis for the observation number, figure 2.

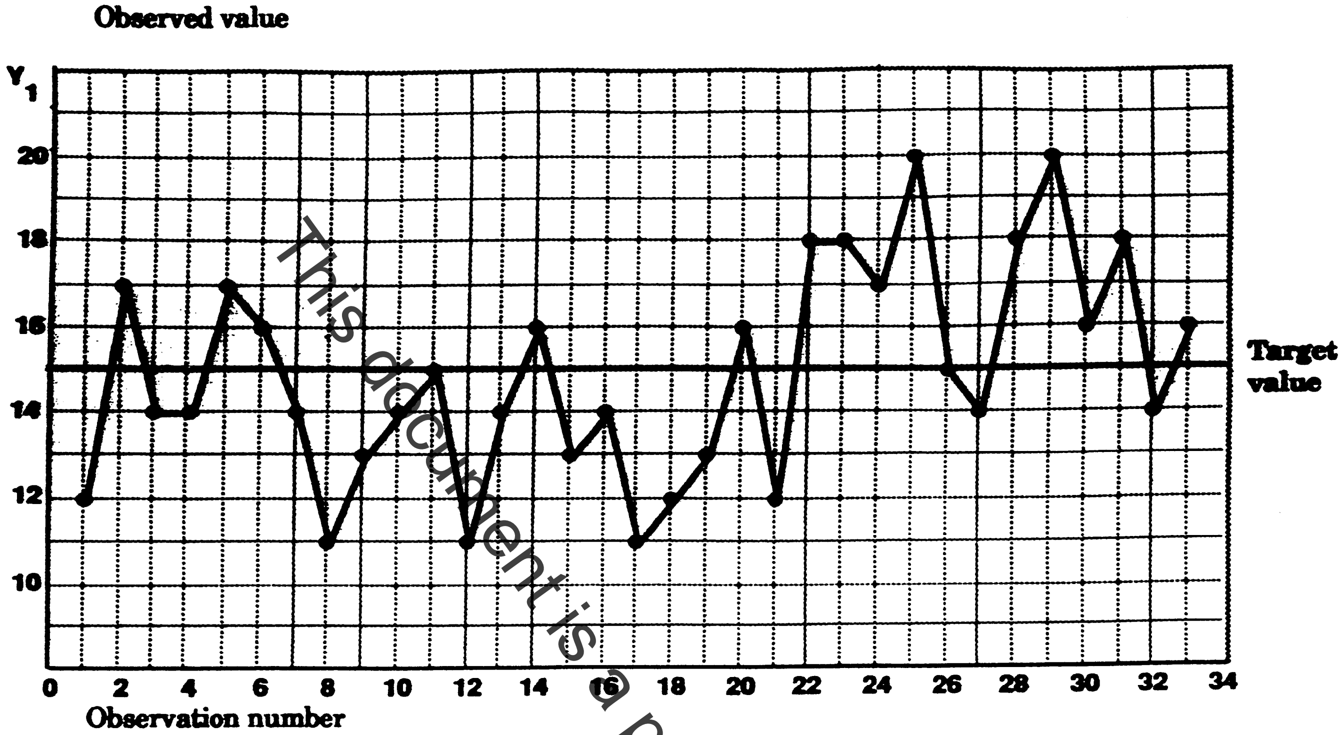

Figure 1 : Conventional chart of data from table 1

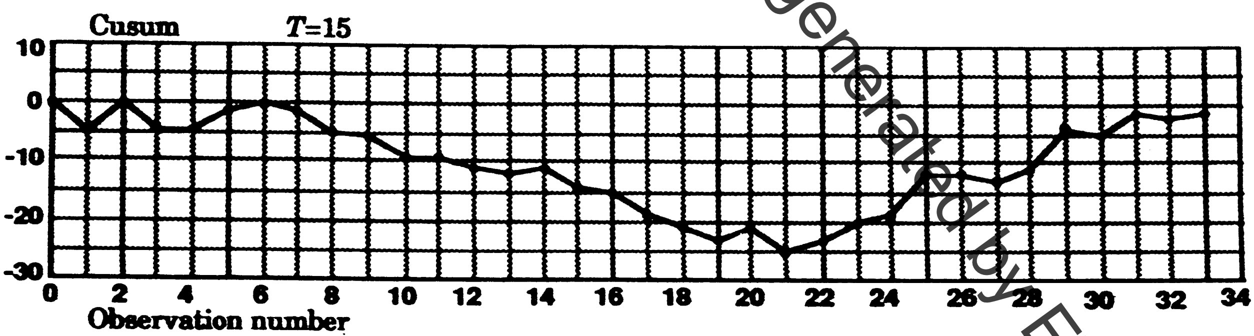

Figure 2 : Cusum chart of data from table 1

The cusum chart clearly separates into three segments. From observations numbered 1 to 7 (inclusive) the cusum path is generally parallel to the observation number axis, i.e. the path is roughly horizontal. From observations 8 to 21 inclusive the path is downward (despite local irregularities such as at observations 14, 20). From observations 22 to 33, the path is upward (again with local irregularities).

Thus it could tentatively be inferred that :

a) observations 1 to 7 constitute a sample from a "population" whose mean is at or near the target value (15) ;

b) observations 8 to 21 appear to have been sampled from a population whose mean is below 15 ;

c) observations 22 onward appear to come from a population whose mean is greater than 15.

There are now a number of questions that might be asked :

1) in the light of the underlying variability (as indicated, for example, by the irregularities in the cusum path) can it be concluded that the changes in slope represent real shifts in average rather than merely lucky or unlucky runs of samples from a stable population ?

2) if the changes are real, how should the data be used to estimate local averages ?

3) to what extent might the inferences or estimates be affected by the choice of the reference value or the cusum scale factor ? Thus figures 3 and 4 show the same series plotted first with the same cusum scale but with a target of 12, and second with a target of 15 but a compressed cusum scale.

In figures 3 and 4, the change in slope around observation number 8 is less apparent. The change around number 21 is still visible, but it is less easy to “pinpoint” in figure 4. Thus the choice of reference value and scale factor need careful attention, to avoid either the suppression of useful information or, conversely, the exaggeration of spurious effects. It is also clear from figure 3 that use of a an inappropriate target value may result in the chart running off the upper of lower edge of the graph paper, although this problem may also be minimized by replotting from a new zero at any point in the sequence.

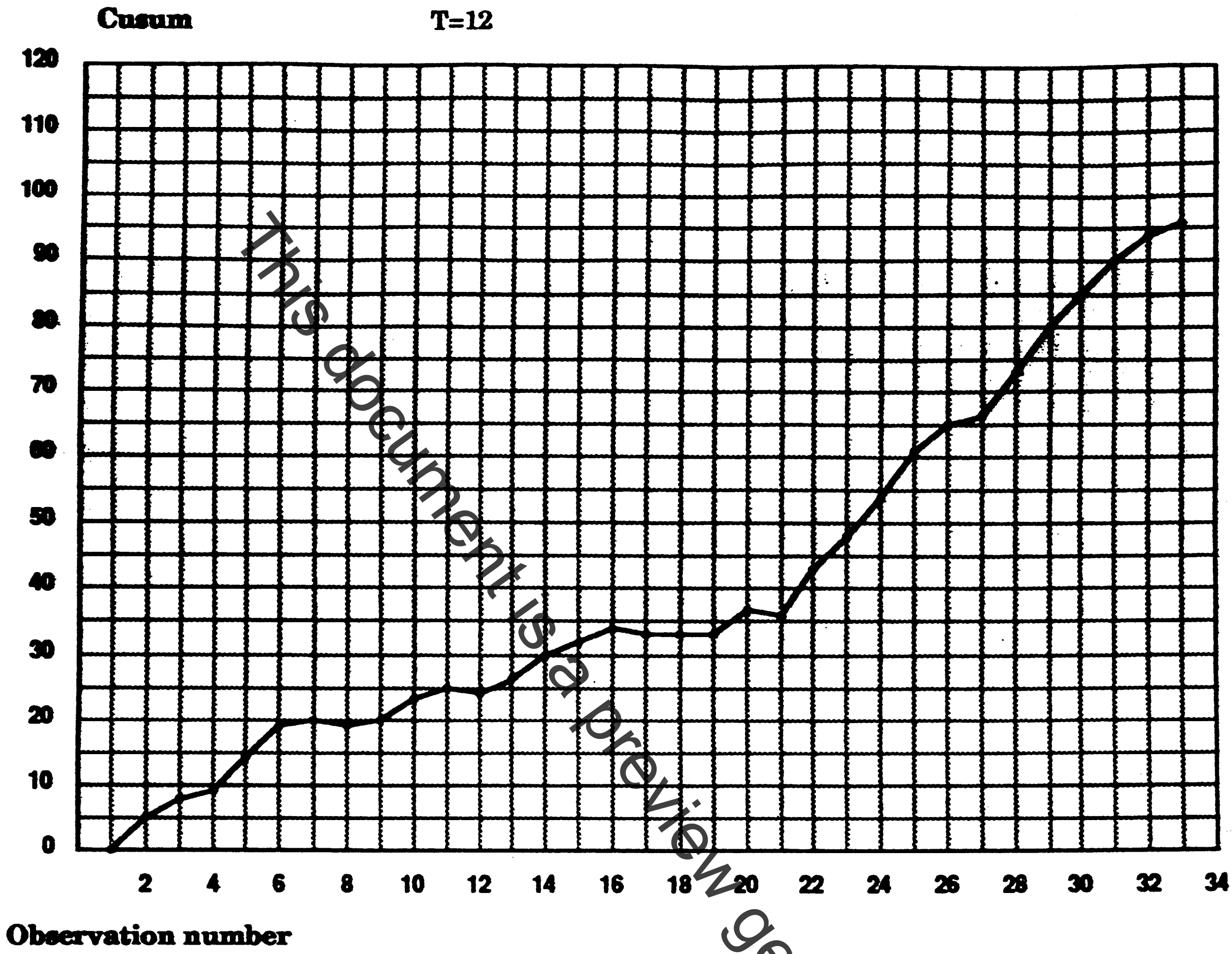

Figure 3 : Cusum chart of data from table 1, with reference value 12

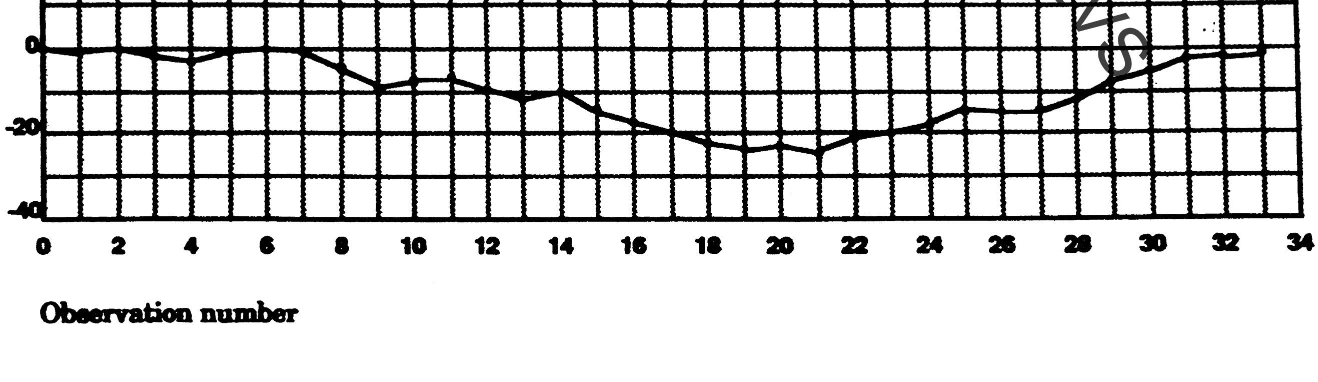

Figure 4 : Cusum chart of data from table 1, with reference value 15 but compressed cusum scale

Cumulative sum charts — Guide to quality control and data analysis using CUSUM techniques

1 Scope and general principles

1.1 General

This standard introduces the principles of cusum charting and includes guidance on the preparation and interpretation of cusum charts using basic decision rules.

1.2 Fundamental requirements

The fundamental requirements from cusum charting are as follows :

a) the observations should be at least on an interval scale of measurement ;

b) there should be logical grounds for the sequence for plotting. This arises naturally in process control.

These requirements are taken in order. The interval property requires any given numerical difference between two observations to have the same interpretation throughout the range of the variable. Thus a difference of 0,1 mm between the lengths of two objects has the same meaning whether the objects are woodscrews of length 10,1 mm and 10,0 mm or steel girders of length 10 000,1 mm and 10 000,0 mm although the latter difference may be unimportant. Many arbitrary scales do not have this property : ratings are an example, where perhaps a serious nonconformity scores 10 points, a moderate nonconformity 5 and minor nonconformity 1. We cannot then interpret this to mean that the following items are necessarily equally undesirable, although their score differences are zero :

- item A one serious nonconformity	Score = 10 ;
- item B two moderate nonconformities	Score = 10 ;
- item C one moderate, five minor nonconformities	Score = 10 ;
- item D ten minor nonconformities	Score = 10.

Interpretation of "average" score could be misleading if the balance of serious, moderate and minor nonconformities, rather than merely their overall frequency, changes.

The logical sequence property may arise in numerous ways. The observations may occur in a time or length sequence, thus forming a natural progression. Monitoring for quality or process control provides many cases of this kind.