



**International
Standard**

ISO 21771-1

**Cylindrical involute gears and
gear pairs —**

**Part 1:
Concepts and geometry**

*Roues et engrenages cylindriques en développante de cercle —
Partie 1: Concepts et géométrie*

**First edition
2024-10**

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO document should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 1, *Nomenclature and wormgearing*.

This first edition of ISO 21771-1 cancels and replaces ISO 21771:2007, which has been technically revised.

The main changes are as follows:

- the sign convention for internal gears used in the ISO 6336 series^[8] has been adopted. The negative value for the number of teeth of an internal gear is applied to the diameters and centre distance, so these dimensions of internal gears have negative values;
- flank direction has been renamed as hand of helix and sign (+/-) of helix angle is used;
- a definition of normal surface has been added and this is used rather than normal plane;
- the annex on tooth thickness was removed because it is now addressed in ISO 21771-2.

Additional material has been added to cover:

- calculation of form diameters for tooth tip corner radius and tooth root fillet radius in the transverse plane for an involute cylindrical gear ([Clauses 10, 11](#) and [Annex B](#));
- calculation of the tooth tip corner radius for a specified form diameter and tip diameter of an involute cylindrical gear;
- calculation of a radius tangent to the involutes of adjacent teeth at root or tip diameter ([Annex A](#));
- generated tooth root fillet shape for individual involute cylindrical gears ([Annex B](#));
- concepts and parameters for involute cylindrical gear pairs with crossed axes ([Clause 6](#) and [Annex C](#));
- geometry of surfaces in contact ([Annex D](#));
- projection of a transverse plane profile of a tooth onto another plane ([Annex E](#));

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— interface to ISO 10828 for involute worm gear geometry ([Annex F](#)).

A list of all parts in the ISO 21771 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

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Cylindrical involute gears and gear pairs —

Part 1: Concepts and geometry

1 Scope

This document specifies the geometric concepts and parameters for cylindrical gears with involute helicoid tooth flanks. Flank modifications are included. The formulae in this document apply to all pressure angles.

It also covers the concepts and parameters for involute cylindrical gear pairs with parallel or crossed axes, and a constant gear ratio. Gear and mating gear in these gear pairs have the same basic rack tooth profile.

2 Normative references

There are no normative references in this document.

3 Terms, definitions, symbols, subscripts and units

3.1 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

3.1.1

basic rack tooth profile for involute gear teeth

tooth profile of a normal section through the teeth of a basic rack which corresponds to an external spur gear with number of teeth $z = \infty$ and diameter $d = \infty$

Note 1 to entry: The tooth of the basic rack tooth profile is bounded by the tip line at the top and by the parallel root line at the bottom. The fillet between the straight part of the profile and the root line is a circular arc with a radius equal to ρ_{fp} .

3.1.2

counterpart rack tooth profile

rack tooth profile symmetrical to the basic rack tooth profile about the datum line P-P and displaced by half a pitch relative to it

3.1.3

nominal involute flank

pure involute flank prior to any modifications

Note 1 to entry: See [4.4](#) for more information on a gear tooth involute flank.