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**Industrial automation systems and  
integration — Product data representation  
and exchange —**

Part 50:

**Integrated generic resource: Mathematical  
constructs**

*Systèmes d'automatisation industrielle et intégration — Représentation  
et échange de données de produits —*

*Partie 50: Ressources génériques intégrées: Constructions mathématiques*



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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75% of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 10303-50 was prepared by Technical Committee ISO/TC 184, *Industrial automation systems and integration*, Subcommittee SC4, *Industrial data*.

This International Standard is organized as a series of parts, each published separately. The structure of this International Standard is described in ISO 10303-1.

Each part of this International Standard is a member of one of the following series: description methods, implementation methods, conformance testing methodology and framework, integrated generic resources, integrated application resources, application protocols, abstract test suites, application interpreted constructs, and application modules. This part is a member of the integrated generic resources series. The integrated generic resources and the integrated application resources specify a single conceptual product data model.

A complete list of parts of ISO 10303 is available from the Internet:

<http://www.nist.gov/sc4/editing/step/titles/>

Annexes A and B form a normative part of this part of ISO 10303. Annexes C and D are for information only.

## Introduction

ISO 10303 is an International Standard for the computer-interpretable representation of product information and for the exchange of product data. The objective is to provide a neutral mechanism capable of describing products throughout their life cycle. This mechanism is suitable not only for neutral file exchange, but also as a basis for implementing and sharing product databases, and as a basis for archiving.

This part of ISO 10303 is a member of the integrated resource series. This part of ISO 10303 specifies the **mathematical\_functions\_schema**.

This part of ISO 10303 specifies EXPRESS data representations for a large class of mathematical functions, expressions, and arrays. They are intended to be used to communicate product property data and related engineering analysis data. Familiarity with the branches of mathematics commonly used in engineering applications is assumed. The central concept is that mathematical functions and arrays are the abstract data objects of interest for expressing any property which requires more than a single number as a value, for describing deterministic relationships between properties, and for documenting behavioural responses of products to varying conditions.

The relationships of the schema in this part of ISO 10303 to other schemas that define the integrated resources of this International Standard are illustrated in Figure 1 using the EXPRESS-G notation. EXPRESS-G is defined in annex D of ISO 10303-11. The documents containing the specifications for these related schemas are identified in note 1 at the beginning of clause 4.

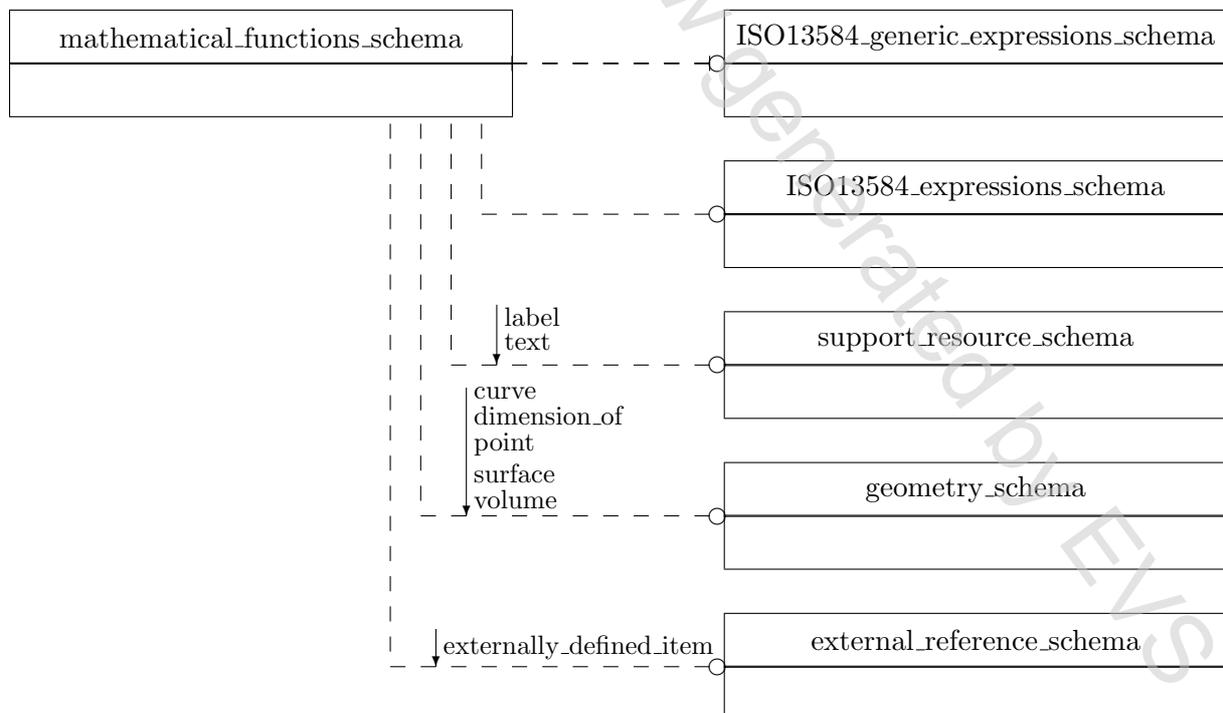


Figure 1 – Schema relationships of the **mathematical\_functions\_schema**



# Industrial automation systems and integration — Product data representation and exchange — Part 50 : Integrated generic resource: Mathematical constructs

## 1 Scope

This part of ISO 10303 specifies the resource constructs for the explicit representation of mathematical structures and data related to properties of a product.

The following are within the scope of this part of ISO 10303:

- multi-dimensional tables;
- mathematical expressions;
- mathematical functions;
- mathematical spaces.

The following are outside the scope of this part of ISO 10303:

- context of application;
- physical units;
- non-mathematical semantics.

## 2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this part of ISO 10303. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this part of ISO 10303 are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO/IEC 8824-1:1998, *Information technology — Abstract Syntax Notation One (ASN.1): Specification of basic notation*

ISO 10303-1:1994, *Industrial automation systems and integration — Product data representation and exchange — Part 1: Overview and fundamental principles*

## ISO 10303-50:2002(E)

ISO 10303-11:1994, *Industrial automation systems and integration — Product data representation and exchange — Part 11: Description methods: The EXPRESS language reference manual*

ISO 10303-41:2000, *Industrial automation systems and integration — Product data representation and exchange — Part 41: Integrated generic resource: Fundamentals of product description and support*

ISO 10303-42:2000, *Industrial automation systems and integration — Product data representation and exchange — Part 42: Integrated generic resource: Geometric and topological representation*

ISO 13584-20:1998, *Industrial automation systems and integration — Parts Library — Part 20: Logical resource: Logical model of expressions*

## 3 Terms, definitions, and symbols

### 3.1 Terms defined in ISO 10303-1

For the purpose of this part of ISO 10303, the following terms defined in ISO 10303-1 apply:

— integrated resource

### 3.2 Other terms and definitions

For the purposes of this part of ISO 10303, the following definitions apply:

#### 3.2.1

##### **actual function domain**

mathematical space containing precisely the tuples of inputs to the function which are related to some tuple of outputs

NOTE See the definitions for “mathematical function” and “function domain”. See also 4.2.1 and 4.2.6.

#### 3.2.2

##### **actual function range**

mathematical space containing precisely the tuples of outputs from the function which are related to some tuple of inputs

NOTE See the definitions for “mathematical function” and “function range”. See also 4.2.1 and 4.2.6.

#### 3.2.3

##### **array function**

function whose domain is either a Cartesian product of finite intervals of integers or the one-tuples from such a Cartesian product

NOTE An element of such a domain is a subscript tuple. Evaluation of the function at such an element produces the “array entry” for that subscript tuple.

#### 3.2.4

##### **bound variable**

variable which has been specifically referenced by a quantifier in a quantifier expression